Solution By G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia; and F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let W=weight of wheel and axle, β=the angle between P and Q and also between P and W since Q and W are parallel. The resultant of P, Q, W due to friction is $\pm\sqrt{(Q+W)^2+P^2+2P(Q+W)\cos\beta}\times\rho\sin\varepsilon$.

$\therefore PR=Qr\pm\rho\sin\varepsilon\sqrt{(Q+W)^2+P^2+2P(Q+W)\cos\beta}$.

When $\beta=0$ this becomes $PR=Qr\pm\rho\sin\varepsilon(P+Q+W)$.

$\therefore P(R\mp\rho\sin\varepsilon)=Q(r\pm\rho\sin\varepsilon)+W\rho\sin\varepsilon$.

Also solved by *ALFRED HUME*.

22. **Proposed by DE VOLSON WOOD, C. E., Professor of Mechanical and Electrical Engineering in Stevens Institute of Technology, Hoboken, New Jersey.**

A prismatic bar having a uniform angular velocity ω and a linear velocity of v feet per second, suddenly snaps (by the disappearance of the cohesive force) into an indefinite number of equal parts; required the resultant angular velocity of each piece and the locus of the parts at the end of t seconds after rupture.

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi.

Take O, the center of gravity of the bar AB, as the origin of a system of rectangular axes, the Y—axis coinciding with the direction of the motion of translation.

Let the motion of rotation be contrary to that of the hands of a clock.

Let the length of the bar be $2nl$, n being the number of equal parts into which it snaps; and let the cross-section and the density, each, be unity.

Denote the middle point of DE, any one of these equal parts, by C, any other point of DE by P.

Let $OC=R$, $OP=r$, and $\angle\times OP=\theta$.

At the instant of separation P has a velocity, v, parallel to Y and a velocity, $r\omega$, perpendicular to OB.

The subsequent motion of DE may be determined by supposing it initially at rest and acted upon by such impulsive forces as are expressed in the actual motion at the instant under consideration.

The element of mass at P is acted upon by an impulsive force parallel to Y measured by the momentum $v.dr$, and by a force perpendicular to OB measured by $r\omega.dr$.

Therefore, taking moments about C, the angular velocity of DE, given by the ratio of the moment of the momentum to the moment of inertia, is

$$\frac{\int[v.\cos\theta+r\omega](r-R)dr}{\frac{2}{3}l^3}$$ the limits being $R+l$ and $R-l$.

Integrating between these limits, the numerator of this fraction becomes $\frac{2}{3}\omega l^3$.

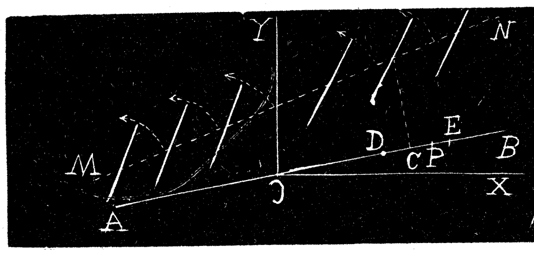
Hence, after separation, DE will rotate about C with an angular velocity equal to that of the original bar.

C, itself, will move in the direction OY with a velocity $v+Rw.\cos\theta$ and in the direction XO with a velocity $R\omega.\sin\theta$.

At the end of t seconds the co-ordinates of C will be given by $x=R\cos\theta-R\omega.\sin\theta.t$ and $y=R\sin\theta+(v+R\ \omega.\cos\theta t)$.

Eliminating R, $y=\dfrac{\tan\theta+\omega t}{1-\omega t.\tan\theta}\cdot x+vt$, or $y=\tan(\theta+\tan^{-1}\omega t)x+vt$.

The locus of the centers of gravity of the parts t seconds after rupture is, therefore, a straight line inclined to X at an angle $\theta+\tan^{-1}\omega t$, and cutting Y at a distance vt from O.



This line coincides with Y after $\dfrac{\cot\theta}{\omega}$ seconds.

When $t=\infty$, the locus is perpendicular to AB.

In the figure the line MN represents the locus, and the arrows the direction of rotation of the parts. The center of gravity of each part moves uniformly in a straight line forever, while the part rotates uniformly about this center of gravity.

This problem was also solved by *F. P. MATZ*.

PROBLEMS.

29. Proposed by J. A CALDERHEAD, A. B., Superintendent of Schools, Limaville, Ohio.

Show that if a body be projected from the angle A of a plane triangle ABC so as to strike the side CB at a point D, then, if its course after reflection at D be parallel to AB, $\tan DAB=\dfrac{(1+\varepsilon)\cot B}{(1-\varepsilon)\cot^2 B}$.

30. Proposed by WILLIAM HOOVER A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

P is the lowest point on the rough circumference of a circle in a vertical plane at which a partical can rest, friction being equal to the pressure; to find the inclination of the radius through P to the horizon.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

23. Proposed by J. M. COLAW, A. M., Principal of High School, Monterey, Virginia.

Find three positive integral numbers such that the product of the first and the sum of the others is a square and the sum of their cubes is a square.